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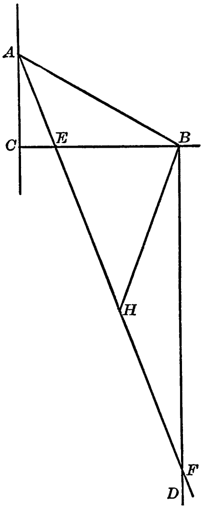
Also solved by E. H. Vance, J. B. Reynolds, and William Hoover.

513. Proposed by ALBERT A. BENNETT, University of Texas.

The following construction for angle trisection was given some years ago in a non-mathematical journal. Let ABC be a right triangle with AB as hypotenuse. Let BD be a ray drawn parallel to AC and extending in the same direction. Let AEF be a variable ray meeting the segment BC in E and the ray BD in F. Show, by elementary methods, that when the variable ray is so adjusted that $EF = 2AD$ then $\angle EAC = \frac{1}{3} \angle BAC$.



Solution by Jos. B. Reynolds, Lehigh University.

I take it that the reading should be $EF = 2AB$. Let the figure be constructed according to the given directions. Join B to H, the middle point of EF. Now $BH = HE = HF = AB$ so that the triangles ABH and BHF are isosceles and, therefore,

$$\angle BAE = \angle BHA = 2 \angle BFH = 2 \angle EAC.$$

Hence,

$$\angle EAC = \tfrac{1}{3} \angle BAC.$$

Also solved by S. W. Reaves and Horace Olson.

MECHANICS.

337. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Assuming that a train may be accelerated by the application of a force equal to 1/40 of its gross weight and be braked with a force equal to 1/10 of its gross weight, show that the least time in which it may run from one to another of two stopping stations 5,000 feet apart is 2 minutes and 5 seconds. Also find the greatest speed during the run to be 54-6/11 miles per hour.

Solution by Captain Stuart C. Godfrey, West Point, N. Y.

A force applied to a body equal to one fortieth of its weight will produce an acceleration of $g/40 = 32/40 = 4/5$ ft. per sec^2.

Then $\alpha_1 = 4/5$ = increasing acceleration during first period of train's motion.

$\alpha_2 = 32/10 = 16/5$ = decreasing acceleration during final period of train's motion.

Let t_1 and t_2 represent the corresponding time intervals, and let $V_1 = \alpha_1 t_1 = \alpha_2 t_2$ = maximum speed attained.

Since $\alpha_2 = 4\alpha_1$, $t_2 = 1/4t_1$. Also, since $s = 1/2\alpha t^2$, the distance corresponding to the final period is

$$\tfrac{1}{2}\alpha_2 t_2^2 = \frac{1}{2}\left(4\alpha_1 \cdot \frac{t_1^2}{4^2}\right) = \tfrac{1}{4}(\tfrac{1}{2}\alpha_1 t_1^2) = \tfrac{1}{4}, \text{ of the distance for the first period.}$$

Now if the reasonable assumption be made that the speed of the train continues to accelerate until the moment when the brakes must be applied to bring it to a stop at the final station, the problem is readily solved without using the maxima and minima method.

For the distances are obviously 4,000 feet and 1,000 feet.

$$t = \sqrt{\frac{2s_1}{\alpha_1}} = \sqrt{\frac{8000 \cdot 5}{4}} = 100 \text{ secs.}$$

$$t_2 = \frac{t_1}{4} = 25 \text{ secs.} \quad \textit{Total time} = \textit{2 min. 5 secs.}$$

Also $V_1 = \alpha_1 t_1 = 4/5 \cdot 100 = 80$ ft. per sec. = 54-6/11 miles per hour.

More rigorously, it is conceivable that the train might retain its maximum speed V_1 for a period t_3 before the brakes were applied. Then, letting $4s$ and s represent the distances corresponding to the periods t_1 and t_2, the time function in terms of s becomes (since $t = \sqrt{2s/\alpha}$)

$$t = \sqrt{\frac{8s}{\alpha_1}} + \frac{1}{4}\sqrt{\frac{8s}{\alpha_1}} + \frac{5000 - 5s}{V_1} = \frac{5}{4}\sqrt{10s} + \frac{5000 - 5s}{\frac{4}{5}\sqrt{10s}} = \frac{5}{4} \cdot \frac{10s + 5000 - 5s}{\sqrt{10s}}.$$

$$\frac{dt}{ds} = \frac{5}{4} \cdot \frac{25s - 25000}{(10s)^{3/2}} = 0.$$

Hence, $s = 1{,}000$ feet, $4s = 4{,}000$ feet, and the distance travelled at uniform speed is zero.

In this case, t is a decreasing function of s until s reaches the critical value 1,000; but it can only become an increasing function thereafter by 5,000—$5s$ becoming negative, which makes the third term of t, and consequently t itself, no longer represent possible physical conditions. The function itself, then, has a true minimum state, but this is an end point of the curve, so far as its physical interpretation is concerned.

Also solved by O. S. ADAMS, CHRISTIAN HORNUNG, FRANK IRWIN, HORACE OLSON, and G. PAASWELL.

338. Proposed by J. B. REYNOLDS, Lehigh University.

A comet in a parabolic orbit crosses the earth's orbit (assumed circular) so that it remains a maximum time within it; find the comet's maximum velocity in miles per second and its time within the earth's orbit in years.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let ρ be the distance at any moment from the sun, p the perpendicular from the sun upon the tangent to the comet's path when the comet is at the distance ρ, then, $4a$ being by the problem the latus rectum of the parabolic path of the comet, we have

$$p^2 = a\rho. \tag{1}$$

Also let F be the attractive force of the sun, h the double area generated by ρ in a unit of time, and θ the angular coördinate corresponding to ρ.

From the theory of central forces,

$$F = \frac{h^2}{p^3}\frac{dp}{d\rho}. \tag{2}$$

From (1),

$$\frac{2}{p^3}\frac{dp}{d\rho} = \frac{1}{a\rho^2}\cdots, \tag{3}$$

and this in (2) gives

$$F = \frac{h^2}{2a\rho^2}. \tag{4}$$

Let $F = \mu$ when $\rho = 1$; then (4) gives

$$h = \sqrt{2a\mu}, \tag{5}$$

(4) then becoming

$$F = \frac{\mu}{\rho^2}. \tag{6}$$

For the velocity v of the earth,

$$v^2 = rF = \frac{\mu}{r}, \tag{7}$$

ρ in (6) being now the earth's distance from the sun, *i. e.*, r. Then $\theta r \div v = \theta r^{3/2}/\sqrt{\mu}$ = the time required by the earth to describe the arc of its circular orbit subtending the angle θ at the sun.

For the comet,

$$dt = \frac{\rho^2 d\theta}{h}, \tag{8}$$

and the polar equation of its orbit is

$$\rho = \frac{2a}{1 + \cos\theta} = \frac{a}{\cos^2\frac{\theta}{2}}. \quad \text{Hence, } \rho^2 d\theta = \frac{a^2 d\theta}{\cos^4\frac{\theta}{2}}, \tag{9}$$

and (8) becomes

$$t = \frac{2a^2}{h}\int\frac{d\theta}{\cos^4\frac{\theta}{2}} = \frac{2a^2}{\sqrt{2a\mu}}\left(\tan\frac{\theta}{2} + \tfrac{1}{3}\tan^3\frac{\theta}{2}\right). \tag{10}$$